



# MTH 301 - Fall 2023

## Introduction to Number Theory

### Course Syllabus

## Course information

**Room:** BH 316

**Time:** MWF 9:10 - 10:00 am

**Credits:** 3.0

**Prerequisites:** A grade of C or better in MTH 220, or permission of the instructor.

**Description:** This course is an introduction to elementary number theory. One of the main driving questions for this course is: how can we find integer solutions of an algebraic equation with integer coefficients? This will motivate the study of several topics including: properties of integers (e.g. divisibility, greatest common divisors, primes, factorizations), modular arithmetic (e.g. linear congruences, Chinese remainder theorem), and quadratic equations (e.g. Legendre symbols, quadratic reciprocity).

**Learning outcomes:** Successful MTH 301 students will be able to:

- define and recognize properties of integers and modular integers, and correctly compute with them;
- solve linear Diophantine equations, and linear and quadratic congruences;
- state and prove foundational results of elementary number theory.

**Online resources:** Visit Blackboard for up-to-date information about this course.

## Instructor

**Name:** Federico Galetto (he/him)

**E-mail:** [f.galetto@csuohio.edu](mailto:f.galetto@csuohio.edu)

**Office:** Rhodes Tower 1219

**In-person office hours:** MWF 10:10 - 11:10 am

**Virtual office hours:** Schedule an appointment at <https://math.galetto.org/appt>

## Materials

Notes and other materials will be distributed on Blackboard. All course materials are copyrighted and cannot be distributed without the permission of the authors.

The following textbooks are recommended. Class notes will follow the outline in the first source, which is on reserve at the library; the second one is available as a free PDF.

### Recommended textbooks:

- Martin H. Weissman, *An Illustrated Theory of Numbers*, AMS, 978-1-4704-3493-9, [https://researchguides.csuohio.edu/er.php?course\\_id=100019](https://researchguides.csuohio.edu/er.php?course_id=100019)
- Karl-Dieter Crisman, *Number Theory: In Context and Interactive*, Available at <http://math.gordon.edu/ntic>.

# Grading

**Grade calculation:** All grades will be posted on Blackboard. Your percentage grade will be computed according to the following breakdown and converted to a letter grade as indicated below.

15%	Homework	A	93%-105%	C+	76%-79%
35%	Quizzes (x7)	A-	90%-92%	C	70%-75%
50%	Tests (x2)	B+	87%-89%	D	60%-69%
100%	Total	B	83%-86%	F	0%-59%
		B-	80%-82%		

## Homework:

- Homework will be assigned on Blackboard and announced in class.
- All assignments must be submitted online by the due date.
- You are expected to work independently on all assignments (see policy on academic integrity).
- The following criteria will be considered in the evaluation of your work.
  - Completeness (answers are provided for all problems, reasoning behind answers is fully justified).
  - Correctness (provided answers are correct, reasonings are sound).
  - Quality of writing (writing is legible and clearly organized, has proper grammar and correct spelling, contains full sentences, uses mathematical notation correctly).
  - Originality (answers and writing are not a mere copy or rewriting of other's work but show additional independent insight).
- The lowest assignment will be dropped.

**Quizzes:** Seven quizzes will be held in class on dates TBA. The quizzes will cover the following topics. References are from Martin H. Weissman, *An Illustrated Theory of Numbers*, AMS, 978-1-4704-3493-9.

- Solutions of a linear Diophantine equation in two variables (Problem 1.15, page 33)
- Solutions of a linear congruence in one variable (Problem 5.19, page 137, general solution in notes)
- Multiplicative inverses in modular arithmetic (notes)
- Large powers via repeated squaring and Euler's theorem (Problem 6.19, page 161; Problem 7.11, page 180; Problem 6.15, page 159)
- Chinese Remainder Theorem type congruences (Problem 7.3, page 176)
- Legendre symbols via Quadratic Reciprocity (Problem 8.29, page 216)
- Solubility of quadratic congruences (notes)

**Tests:** There will be two tests held in class on the following (tentative) dates:

- Wednesday, October 11, 2023
- Wednesday, November 29, 2023

During a test, you may be asked to:

- define and provide examples for concepts introduced in class;
- state and prove theorems covered in class;
- discuss applications of the theory.

You will be required to show identification.

**Final exam:** An optional final exam will be held on Friday, December 15, 2022, 8 - 10 am in BH 316. You will be required to show identification. As part of the final exam, you may retake one (your lowest) or both tests, and your lowest or lowest two quizzes. The new scores will be officially recorded only if they are higher than the original ones; otherwise, you will retain your original score.

**Participation:** Active participation is encouraged and can contribute positively to your grade. Such participation includes asking good questions during class, answering questions in class, contributing to class discussions and activities, etc.

**Grade appeal:** You are responsible for checking feedback on your work, and for ensuring evaluations are reported correctly on Blackboard. Any appeal request must be submitted within one week after a grade is posted.

## Policies

**Communication:** All communication, outside of class meetings, will be conducted via email. You can reach me at [f.galetto@csuohio.edu](mailto:f.galetto@csuohio.edu). I strongly recommend using your CSU email account, as email from other accounts is often marked as spam. You can usually expect to receive an answer from me within 24 hours Monday through Friday, or within 48 hours during the weekends and festivities.

**Attendance:** You are expected to attend all classes. If you are unable to attend a class, it is your responsibility to notify your instructor in advance and to inquire about any topics covered and announcements made during that class.

**Electronic devices:** The use of electronic devices such as (but not limited to) phones, smartwatches, computers, tablets, and headphones is prohibited during class, unless otherwise required (e.g. to connect to a Zoom meeting) or indicated by the instructor.

**Excused absences:** You may only be excused from class and class related activities in case of university sanctioned activities (such as conferences, competitions, etc.) or in case of medical conditions. With the exception of unforeseen medical emergencies, you must notify your instructor of your absence and present sufficient documentation in advance. Sufficient documentation includes an invitation to attend an event or a doctor's note indicating you cannot attend on the scheduled date.

**Extra credit and makeups:** There is no extra credit in this class. There are no makeups in this class other than for excused absences. Insofar as circumstances allow, all makeups have to be arranged in advance.

**Academic integrity:** Cheating and/or plagiarism will not be tolerated. Cheating includes copying or receiving help from another student on quizzes, tests or exams, as well as allowing another student to copy from your work. Receiving help from someone else by using an electronic device such as a mobile phone or a smartwatch constitutes cheating. Copying another student's homework, or allowing someone else to do your homework for you, is also considered cheating. If cheating occurs in a quiz or test, the student will receive a grade of 0 for that component of the course. If a student cheats a second time during the course, the student will receive an F for the course. If cheating occurs on the final exam, the student will receive a grade of F in the course. Any cheating activity may be reported for further action. Information regarding the official CSU Policy on Academic Misconduct can be found at [https://www.csuohio.edu/sites/default/files/3344-21-02\\_0.pdf](https://www.csuohio.edu/sites/default/files/3344-21-02_0.pdf).

**Safe space:** I am committed to making our encounters (in class, during office hours, in person, and online) feel safe for everyone involved, within my ability to do so. All participants are expected to conduct themselves in a respectful manner towards other participants and their ideas. Behaviors that are disrespectful or discriminatory towards other individuals or groups of individuals will not be tolerated.

**Accommodations:** Educational access is the provision of classroom accommodations, auxiliary aids and services to ensure equal educational opportunities for all students regardless of their disability. Students who feel they may need an accommodation based on the impact of a disability should contact the Office of Disability Services at 216-687-2015. The Office is located in BH 147. Accommodations need to be requested in advance and will not be granted retroactively.

**Withdrawals:** The last day to withdraw is Friday, November 3, 2023. Withdrawing from the course may put a student in violation of the federally mandated standards for academic progress (SAP) that a student must maintain to be eligible for financial aid. Please visit <https://www.csuohio.edu/financial-aid/standards-academic-progress-sap> for more information.

**Course modifications:** The instructor retains the right to modify the contents of the course, including grading criteria and course policies. Reasonable notice will be given for all time sensitive matters. Course changes will be communicated in class and on Blackboard.

## Calendar

Here is a rough calendar for the course. Please note this is tentative and subject to change. References are from Martin H. Weissman, *An Illustrated Theory of Numbers*, AMS, 978-1-4704-3493-9.

Weeks	Topics	Reference
1	Division with remainder, divisibility	Chapter 0, pages 12-17
2-3	GCD, Euclidean Algorithm, LCM, linear Diophantine equations	Chapter 1
4-5	Prime numbers and factorizations	Chapter 2, pages 47-50, 56-59, 61
6-7	Modular arithmetic and congruences	Chapter 5, 127-139
8-9	Totient, Euler's Theorem and Fermat's Little Theorem, modular dynamics	Chapter 6, pages 153-159, 161
10-11	Chinese Remainder Theorem, RSA cryptosystem	Chapter 7, pages 173-181, 186-187
12-14	Legendre symbol, Quadratic reciprocity	Chapter 8, pages 194-199, 202-203
15	Catch-up and/or applications	